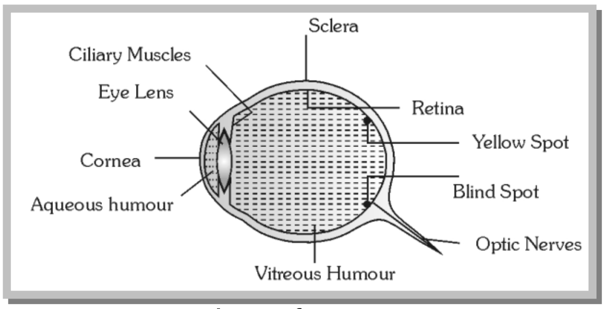
Optical Instruments

Human Eye



(1) **Eye lens :** Over all behaves as a convex lens of $\mu = 1.437$

(2) **Retina :** Real and inverted image of an object, obtained at retina, brain sense it erect.

(3) **Yellow spot :** It is the most sensitive part, the image formed at yellow spot is brightest.

(4) **Blind spot :** Optic nerves goes to brain through blind spot. It is not sensitive for light.

(5) **Ciliary muscles -** Eye lens is fixed between these muscles. It's both radius of curvature can be changed by applying pressure on it through ciliary muscles.

(6) **Power of accomodation :** The ability of eye to see near objects as well as far objects is called power of accomodation.

Note : ❑ When we look distant objects, the eye is relaxed and it's focal length is largest.

(7) **Range of vision :** For healthy eye it is 25 *cm* (near point) to ∞ (far point).

A normal eye can see the objects clearly, only if they are at a distance greater than 25 *cm.* This distance is called Least distance of distinct vision and is represented by *D*.

(8) **Persistence of vision :** Is 1/10 *sec. i.e.* if time interval between two consecutive light pulses is lesser than 0.1 *sec*., eye cannot distinguish them separately.

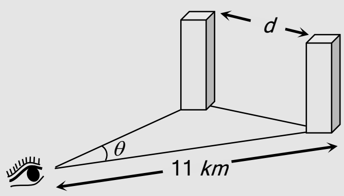
(9) **Binocular vision :** The seeing with two eyes is called binocular vision.

(10) **Resolving limit :** The minimum angular displacement between two objects, so that they are just resolved is called resolving limit. For eye it is $1' = \left(\frac{1}{60}\right)^o$.

Specific Example

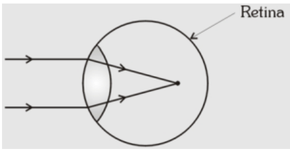
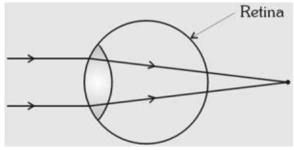
A person wishes to distinguish between two pillars located at a distances of 11 *Km.* What should be the minimum distance between the pillars.

Solution : As the limit of resolution of eye is $\left(\frac{1}{60}\right)^o$

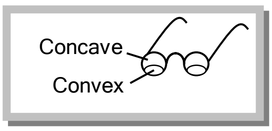


So $\theta > \left(\frac{1}{60}\right)^o \Rightarrow \frac{d}{11 \times 10^3} > \left(\frac{1}{60}\right) \times \frac{\pi}{180} \Rightarrow d > 3.2\,m$

(11) **Defects in eye**

Myopia (short sightness)	Hypermetropia (long sightness)
(i) Distant objects are not seen clearly but nearer objects are clearly visible.	(i) Distant objects are seen clearly but nearer object are not clearly visible.
(ii) Image formed before the retina. 	(ii) Image formed behind the retina. 
(iii) Far point comes closer.	(iii) Near point moves away
(iv) Reasons : (a) Focal length or radii of curvature of lens reduced or power of lens increases. (b) Distance between eye lens and retina increases.	(iv) Reasons : (a) Focal length or radii of curvature of lens increases or power of lens decreases. (b) Distance between eye lens and retina decreases.
(v) Removal : By using a concave lens of suitable focal length.	(v) Removal : By using a convex lens.
(vi) Focal length : (a) A person can see upto distance $\rightarrow x$ wants to see $\rightarrow \infty$, so focal length of used lens $f = -x$ = - (defected far point) (b) A person can see upto distance $\rightarrow x$ wants to see distance $\rightarrow y$ ($y > x$) so $f = \frac{xy}{x-y}$	(vi) Focal length : (a) A person cannot see before distance $\rightarrow d$ wants to see the object place at distance $\rightarrow D$ so $f = \frac{dD}{d-D}$

Presbyopia : In this defect both near and far objects are not clearly visible. It is an old age disease and it is due to the loosing power of accommodation. It can be removed by using bifocal lens.



Astigmatism : In this defect eye cannot see horizontal and vertical lines clearly, simultaneously. It is due to imperfect spherical nature of eye lens. This defect can be removed by using cylindrical lens (Torric lenses).

Microscope

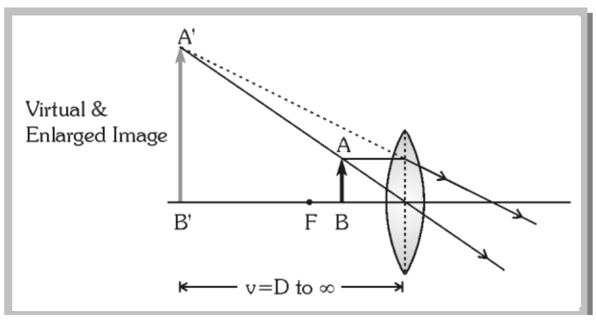
It is an optical instrument used to see very small objects. It's magnifying power is given by

$$m = \frac{\text{Visual angle with instrument } (\beta)}{\text{Visual angle when object is placed at least distance of distinct vision } (\alpha)}$$

(1) **Simple miscroscope**

(i) It is a single convex lens of lesser focal length.

(ii) Also called magnifying glass or reading lens.



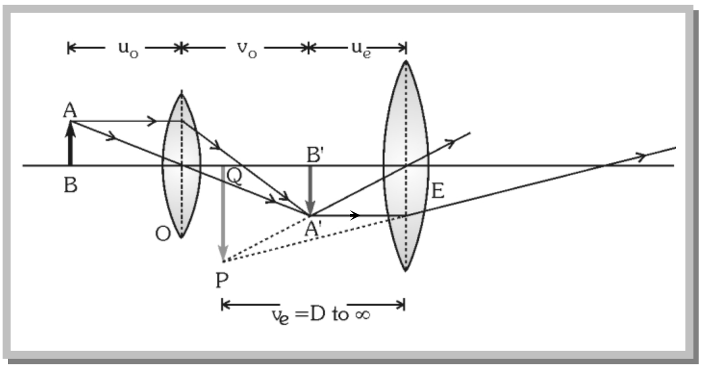
(iii) Magnification's, when final image is formed at *D* and ∞ (*i.e.* m_D and m_∞)

$$m_D = \left(1 + \frac{D}{f}\right)_{max} \text{ and } m_\infty = \left(\frac{D}{f}\right)_{min}$$

Note : ❑ $m_{max.} - m_{min.} = 1$

❑ If lens is kept at a distance *a* from the eye then $m_D = 1 + \frac{D-a}{f}$ and $m_\infty = \frac{D-a}{f}$

(2) **Compound microscope**



(i) Consist of two converging lenses called objective and eye lens.

(ii) $f_{\text{eye lens}} > f_{\text{objective}}$ and (diameter)$_{\text{eye lens}}$ > (diameter)$_{\text{objective}}$

(iii) Final image is magnified, virtual and inverted.

(iv) u_0 = Distance of object from objective (*o*), v_0 = Distance of image ($A'B'$) formed by objective from objective, u_e = Distance of $A'B'$ from eye lens, v_e = Distance of final image from eye lens, f_0 = Focal length of objective, f_e = Focal length of eye lens.

Magnification : $$m_D = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) = -\frac{f_0}{(u_0 - f_0)}\left(1 + \frac{D}{f_e}\right) = -\frac{(v_0 - f_0)}{f_0}\left(1 + \frac{D}{f_e}\right)$$

$$m_\infty = -\frac{v_0}{u_0} \cdot \frac{D}{F_e} = \frac{-f_0}{(u_0 - f_0)}\left(\frac{D}{f_e}\right) = -\frac{(v_0 - f_0)}{f_0} \cdot \frac{D}{F_e}$$

Length of the tube (*i.e.* distance between two lenses)

When final image is formed at *D*; $$L_D = v_0 + u_e = \frac{u_0 f_0}{u_0 - f_0} + \frac{f_e D}{f_e + D}$$

When final images is formed at ∞ ; $$L_\infty = v_0 + f_e = \frac{u_0 f_0}{u_0 - f_0} + f_e$$

(Do not use sign convention while solving the problems)

Note : ❑ $m_\infty = \frac{(L_\infty - f_0 - f_e)D}{f_0 f_e}$

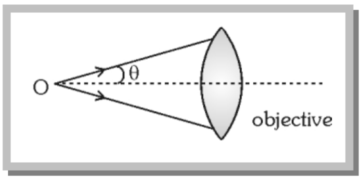
❑ For maximum magnification both f_0 and f_e must be less.

❑ $m = m_{\text{objective}} \times m_{\text{eye lens}}$

❑ If objective and eye lens are interchanged, practically there is no change in magnification.

(3) **Resolving limit and resolving power :** In reference to a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (*RL*) and it's reciprocal is called Resolving power (*RP*)

$$R.L. = \frac{\lambda}{2\mu \sin\theta} \text{ and } R.P. = \frac{2\mu\sin\theta}{\lambda} \Rightarrow R.P. \propto \frac{1}{\lambda}$$



λ = Wavelength of light used to illuminate the object,

μ = Refractive index of the medium between object and objective,

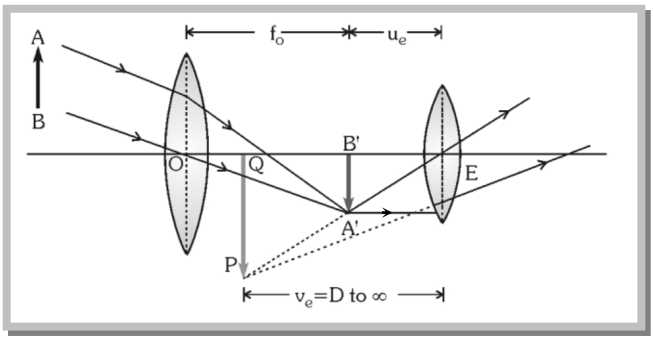
θ = Half angle of the cone of light from the point object, $\mu \sin\theta$ = Numerical aperture.

Note : ❑ Electron microscope : electron beam $(\lambda \approx 1 Å)$ is used in it so it's *R.P.* is approx 5000 times more than that of ordinary microscope $(\lambda \approx 5000\ Å)$

Telescope

By telescope distant objects are seen.

(1) **Astronomical telescope**



(i) Used to see heavenly bodies.

(ii) $f_{\text{objective}} > f_{\text{eyelens}}$ and $d_{\text{objective}} > d_{\text{eye lens}}$.

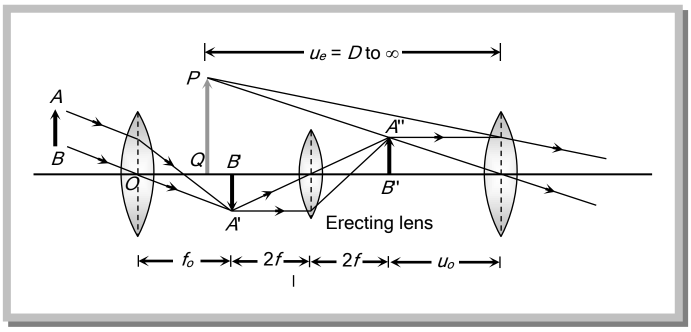
(iii) Intermediate image is real, inverted and small.

(iv) Final image is virtual, inverted and small.

(v) Magnification : $m_D = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$ and $m_\infty = -\frac{f_o}{f_e}$

(vi) Length : $L_D = f_0 + u_e = f_0 + \frac{f_e D}{f_e + D}$ and $L_\infty = f_0 + f_e$

(2) **Terrestrial telescope**



(i) Used to see far off object on the earth.

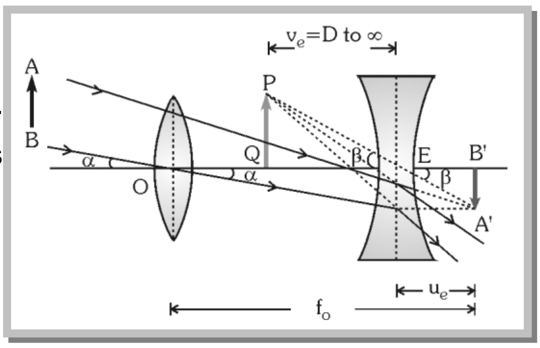
(ii) It consists of three converging lens : objective, eye lens and erecting lens.

(iii) It's final image is virtual erect and smaller.

(iv) Magnification : $m_D = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$ and $m_\infty = \frac{f_0}{f_e}$

(v) Length : $L_D = f_0 + 4f + u_e = f_0 + 4f + \frac{f_e D}{f_e + D}$ and $L_\infty = f_0 + 4f + f_e$

(3) **Galilean telescope**



(i) It is also a terrestrial telescope but of much smaller field of view.

(ii) Objective is a converging lens while eye lens is diverging lens

(iii) Magnification : $m_D = \frac{f_0}{f_e}\left(1 - \frac{f_e}{D}\right)$ and $m_\infty = \frac{f_0}{f_e}$

(iv) Length : $L_D = f_0 - u_e$ and $L_\infty = f_0 - f_e$

(4) **Resolving limit and resolving power**

Smallest angular separations ($d\theta$) between two distant objects, whose images are separated in the telescope is called resolving limit. So resolving limit $d\theta = \frac{1.22\lambda}{a}$

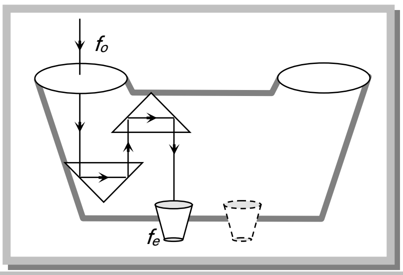
and resolving power $(RP) = \frac{1}{d\theta} = \frac{a}{1.22\lambda} \Rightarrow R.P. \propto \frac{1}{\lambda}$ where a = aperture of objective.

Note : ❑ Minimum separation (d) between objects, so they can just resolved by a telescope is - $d = \frac{r}{R.P.}$

where r = distance of objects from telescope.

(5) **Binocular**

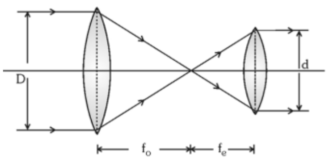
If two telescopes are mounted parallel to each other so that an object can be seen by both the eyes simultaneously, the arrangement is called 'binocular'. In a binocular, the length of each tube is reduced by using a set of totally reflecting prisms which provided intense, erect image free from lateral inversion. Through a binocular we get two images of the same object from different angles at same time. Their superposition gives the perception of depth also along with length and breadth, *i.e.*, binocular vision gives proper three-dimensional (3D) image.



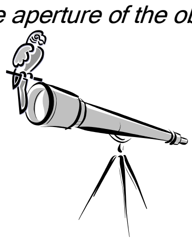
Concepts

- *As magnifying power is negative, the image seen in astronomical telescope is truly inverted, i.e., left is turned right with upside down simultaneously. However, as most of the astronomical objects are symmetrical this inversion does not affect the observations.*
- *Objective and eye lens of a telescope are interchanged, it will not behave as a microscope but object appears very small.*
- *In a telescope, if field and eye lenses are interchanged magnification will change from (f_o / f_e) to (f_e / f_o), i.e., it will change from m to (1/m), i.e., will become (1/m^2) times of its initial value.*
- *As magnification for normal setting as (f_o / f_e), so to have large magnification, f_o must be as large as practically possible and f_e small. This is why in a telescope, objective is of large focal length while eye piece of small.*
- *In a telescope, aperture of the field lens is made as large as practically possible to increase its resolving power as resolving power of a telescope ∝ (D/λ)*. Large aperture of objective also helps in improving the brightness of image by gathering more light from distant object. However, it increases aberrations particularly spherical.*
- *For a telescope with increase in length of the tube, magnification decreases.*
- *In case of a telescope if object and final image are at infinity then :*

$$m = \frac{f_o}{f_e} = \frac{D}{d}$$



- *If we are given four convex lenses having focal lengths $f_1 > f_2 > f_3 > f_4$. For making a good telescope and microscope. We choose the following lenses respectively. Telescope $f_1(o), f_4(e)$ Microscope $f_4(o), f_3(e)$*
- *If a parrot is sitting on the objective of a large telescope and we look towards (or take a photograph)of distant astronomical object (say moon) through it, the parrot will not be seen but the intensity of the image will be slightly reduced as the parrot will act as obstruction to light and will reduce the aperture of the objective.*



Examples

***Example*: 1** A man can see the objects upto a distance of one metre from his eyes. For correcting his eye sight so that he can see an object at infinity, he requires a lens whose power is

or

A man can see upto 100 *cm* of the distant object. The power of the lens required to see far objects will be

[MP PMT 1993, 2003]

(a) +0.5 *D* (b) +1.0 *D* (c) +2.0 *D* (d) -1.0 *D*

Solution: (d) *f* = -(defected far point) = - 100 *cm*. So power of the lens $P = \frac{100}{f} = \frac{100}{-100} = -1D$

***Example*: 2** A man can see clearly up to 3 *metres*. Prescribe a lens for his spectacles so that he can see clearly up to 12 *metres*

[DPMT 2002]

(a) - 3/4 *D* (b) 3 *D* (c) - 1/4 *D* (d) - 4 *D*

Solution: (c) By using $f = \frac{xy}{x-y} \Rightarrow f = \frac{3\times 12}{3-12} = -4m$. Hence power $P = \frac{1}{f} = -\frac{1}{4}D$

***Example*: 3** The diameter of the eye-ball of a normal eye is about 2.5 *cm*. The power of the eye lens varies from

(a) 2 *D* to 10 *D* (b) 40 *D* to 32 *D* (c) 9 *D* to 8 *D* (d) 44 *D* to 40 *D*

Solution: (d) An eye sees distant objects with full relaxation so $\frac{1}{2.5\times 10^{-2}} - \frac{1}{-\infty} = \frac{1}{f}$ or $P = \frac{1}{f} = \frac{1}{25\times 10^{-2}} = 40\,D$

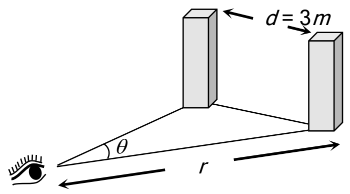
An eye sees an object at 25 *cm* with strain so $\frac{1}{2.5\times 10^{-2}} - \frac{1}{-25\times 10^{-2}} = \frac{1}{f}$ or $P = \frac{1}{f} = 40 + 4 = 44\,D$

***Example*: 4** The resolution limit of eye is 1 *minute*. At a distance of *r* from the eye, two persons stand with a lateral separation of 3 *metre*. For the two persons to be just resolved by the naked eye, *r* should be

(a) 10 *km* (b) 15 *km* (c) 20 *km* (d) 30 *km*

Solution: (a) From figure $\theta = \frac{d}{r}$; where $\theta = 1' = \left(\frac{1}{60}\right)^{o} = \left(\frac{1}{60}\right)\times\frac{\pi}{180}\,rad$

$\Rightarrow 1\times\frac{1}{60}\times\frac{\pi}{180} = \frac{3}{r} \Rightarrow$ *r* = 10 *km*



***Example*: 5** Two points separated by a distance of 0.1 *mm* can just be resolved in a microscope when a light of wavelength 6000 *Å* is used. If the light of wavelength 4800 *Å* is used this limit of resolution becomes

[UPSEAT 2002]

(a) 0.08 *mm* (b) 0.10 *mm* (c) 0.12 *mm* (d) 0.06 *mm*

Solution: (a) By using resolving limit (R.L.) $\propto \lambda \Rightarrow \frac{(R.L.)_1}{(R.L.)_2} = \frac{\lambda_1}{\lambda_2} \Rightarrow \frac{0.1}{(R.L.)_2} = \frac{6000}{4800} \Rightarrow (R.L.)_2 = 0.08\,mm$.

***Example*: 6** In a compound microscope, the focal lengths of two lenses are 1.5 *cm* and 6.25 *cm* an object is placed at 2 *cm* form objective and the final image is formed at 25 *cm* from eye lens. The distance between the two lenses is

[EAMCET (Med.) 2000]

(a) 6.00 *cm* (b) 7.75 *cm* (c) 9.25 *cm* (d) 11.00 *cm*

Solution: (d) It is given that f_o = 1.5 *cm*, f_e = 6.25 *cm*, u_o = 2 *cm*

When final image is formed at least distance of distinct vision, length of the tube $L_D = \frac{u_o f_o}{u_o - f_o} + \frac{f_e D}{f_e + D}$

$\Rightarrow L_D = \frac{2\times 1.5}{(2-1.5)} + \frac{6.25\times 25}{(6.25+25)} = 11\,cm$.

***Example*: 7** The focal lengths of the objective and the eye-piece of a compound microscope are 2.0 *cm* and 3.0 *cm* respectively. The distance between the objective and the eye-piece is 15.0 *cm*. The final image formed by the eye-piece is at infinity. The two lenses are thin. The distances in *cm* of the object and the image produced by the objective measured from the objective lens are respectively **[IIT-JEE 1995]**

(a) 2.4 and 12.0 (b) 2.4 and 15.0 (c) 2.3 and 12.0 (d) 2.3 and 3.0

Solution: (a) Given that $f_o = 2\,cm$, $f_e = 3\,cm$, $L_\infty = 15\,cm$

By using $L_\infty = v_o + f_e \Rightarrow 15 = v_o + 3 \Rightarrow v_o = 12\,cm$. Also $\frac{v_o}{u_o} = \frac{v_o - f_o}{f_o} \Rightarrow \frac{12}{u_o} = \frac{12-2}{2} \Rightarrow u_o = 2.4\,cm$.

***Example*: 8** The focal lengths of the objective and eye-lens of a microscope are 1 *cm* and 5 *cm* respectively. If the magnifying power for the relaxed eye is 45, then the length of the tube is **[CPMT 1979]**

(a) 30 *cm* (b) 25 *cm* (c) 15 *cm* (d) 12 *cm*

Solution: (c) Given that $f_o = 1\,cm$, $f_e = 5\,cm$, $m_\infty = 45$

By using $m_\infty = \frac{(L_\infty - f_o - f_e)}{f_o f_e} \Rightarrow 45 = \frac{(L_\infty - 1 - 5)\times 25}{1\times 5} \Rightarrow L_\infty = 15\,cm$

***Example*: 9** If the focal lengths of objective and eye lens of a microscope are 1.2 *cm* and 3 *cm* respectively and the object is put 1.25 *cm* away from the objective lens and the final image is formed at infinity, then magnifying power of the microscope is

(a) 150 (b) 200 (c) 250 (d) 400

Solution: (b) Given that $f_o = 1.2\,cm$, $f_e = 3\,cm$, $u_o = 1.25\,cm$

By using $m_\infty = -\frac{f_o}{(u_o - f_o)} \cdot \frac{D}{f_e} \Rightarrow m_\infty = -\frac{1.2}{(1.25 - 1.2)} \times \frac{25}{3} = -200$.

***Example*: 10** The magnifying power of an astronomical telescope is 8 and the distance between the two lenses is 54*cm*. The focal length of eye lens and objective lens will be respectively **[MP PMT 1991; CPMT 1991; Pb. PMT 2001]**

(a) 6 *cm* and 48 *cm* (b) 48 *cm* and 6 *cm* (c) 8 *cm* and 64 *cm* (d) 64 *cm* and 8 *cm*

Solution: (a) Given that $m_\infty = 8$ and $L_\infty = 54$

By using $|m_\infty| = \frac{f_o}{f_e}$ and $L_\infty = f_o + f_e$ we get $f_o = 6\,cm$ and $f_e = 48\,cm$.

***Example*: 11** If an object subtend angle of 2° at eye when seen through telescope having objective and eyepiece of focal length $f_o = 60\,cm$ and $f_e = 5\,cm$ respectively than angle subtend by image at eye piece will be **[UPSEAT 2001]**

(a) 16° (b) 50° (c) 24° (d) 10°

Solution: (c) By using $\frac{\beta}{\alpha} = \frac{f_o}{f_e} \Rightarrow \frac{\beta}{20} = \frac{60}{5} \Rightarrow \beta = 24°$

***Example*: 12** The focal lengths of the lenses of an astronomical telescope are 50 *cm* and 5 *cm*. The length of the telescope when the image is formed at the least distance of distinct vision is **[EAMCET (Engg.) 2000]**

(a) 45 *cm* (b) 55 *cm* (c) $\frac{275}{6}$ *cm* (d) $\frac{325}{6}$ *cm*

Solution: (d) By using $L_D = f_o + u_e = f_o + \frac{f_e D}{f_e + D} = 50 + \frac{5\times 25}{(5+25)} = \frac{325}{6}\,cm$

***Example*: 13** The diameter of moon is $3.5\times 10^3\,km$ and its distance from the earth is $3.8\times 10^5\,km$. If it is seen through a telescope whose focal length for objective and eye lens are 4 *m* and 10 *cm* respectively, then the angle subtended by the moon on the eye will be approximately

(a) 15° (b) 20° (c) 30° (d) 35°

Solution: (b) The angle subtended by the moon on the objective of telescope $\alpha = \frac{3.5\times 10^3}{3.8\times 10^5} = \frac{3.5}{3.8}\times 10^{-2}$ *rad*

Also $m = \frac{f_o}{f_e} = \frac{\beta}{\alpha} \Rightarrow \frac{400}{10} = \frac{\beta}{\alpha} \Rightarrow \beta = 40\,\alpha \Rightarrow \beta = 40 \times \frac{3.5\times 10^3}{3.8\times 10^5} \times \frac{180}{\pi} = 20°$

Example: 14 A telescope has an objective lens of 10 *cm* diameter and is situated at a distance one *kilometre* from two objects. The minimum distance between these two objects, which can be resolved by the telescope, when the mean wavelength of light is 5000 Å, is of the order of

(a) 0.5 *m* (b) 5 *m* (c) 5 *mm* (d) 5*cm*

Solution: (b) Suppose minimum distance between objects is *x* and their distance from telescope is *r*

So Resolving limit $d\theta = \frac{1.22\lambda}{a} = \frac{x}{r} \Rightarrow x = \frac{1.22\lambda \times r}{a} = \frac{1.22 \times (5000 \times 10^{-10}) \times (1 \times 10^{3})}{(0-1)} = 6.1 \times 10^{-3} m = 6.1\, mm$

Hence, It's order is ≈ 5 *mm*.

Example: 15 A compound microscope has a magnifying power 30. The focal length of its eye-piece is 5 *cm*. Assuming the final image to be at the least distance of distinct vision. The magnification produced by the objective will be

(a) +5 (b) - 5 (c) +6 (d) - 6

Solution (b) Magnification produced by compound microscope $m = m_o \times m_e$

where m_o = ? and $m_e = \left(1 + \frac{D}{f_e}\right) = 1 + \frac{25}{5} = 6 \Rightarrow 30 = -m_o \times 6 \Rightarrow m_o = -5$.

Tricky example: 1

A man is looking at a small object placed at his least distance of distinct vision. Without changing his position and that of the object he puts a simple microscope of magnifying power 10 *X* and just sees the clear image again. The angular magnification obtained is

(a) 2.5 (b) 10.0 (c) 5.0 (d) 1.0

Solution : (d) Angular magnification $= \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{I/D}{O/D} = \frac{I}{O}$

Since image and object are at the same position, $\frac{I}{O} = \frac{v}{u} = 1 \Rightarrow$ Angular magnification = 1

Tricky example: 2

A compound microscope is used to enlarge an object kept at a distance 0.03*m* from it's objective which consists of several convex lenses in contact and has focal length 0.02*m*. If a lensof focal lengt

(a) 2.5 *cm* (b) 6 *cm* (c) 15 *cm* (d) 9 *cm*

Solution : (d) If initially the objective (focal length F_o) forms the image at distance v_o then $v_o = \frac{u_o f_o}{u_o - f_o} = \frac{3 \times 2}{3-2} = 6\, cm$

Now as in case of lenses in contact $\frac{1}{F_o} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ... = \frac{1}{f_1} + \frac{1}{F_o'}$ $\left\{\text{where } \frac{1}{F_o'} = \frac{1}{f_2} + \frac{1}{f_3} + ...\right\}$

So if one of the lens is removed, the focal length of the remaining lens system

$\frac{1}{F_o'} = \frac{1}{F_0} - \frac{1}{f_1} = \frac{1}{2} - \frac{1}{10} \Rightarrow F_o' = 2.5\, cm$

This lens will form the image of same object at a distance v_o' such that $v_o' = \frac{u_o F_o'}{u_o - F_o'} = \frac{3 \times 2.5}{(3-2.5)} = 15\, cm$

So to refocus the image, eye-piece must be moved by the same distance through which the image formed by the objective has shifted *i.e.* 15 - 6 = 9 *cm*.